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A unique fixation of the locus of basis elements in graphs

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Abstract

The resolving set consisting basis elements is not unique for any graph. This paper examines how the basis elements can be uniquely placed on a graph so as to solve the network flow problems. A relation between connectivity and metric dimension of graph has been verified. Also discussed the parameter changes occurred in molecular graphs due to catenation.

Key words: Metric dimension, cut vertex, connectivity, molecular graph, hydrocarbons

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INTRODUCTION

Connectivity is one of the basic notions of graph theory. It is closely associated with the theory of network problems. Vertex connectivity plays a sufficient role to estimate the capacity of a network's normal operation. It is clear that the graph will become disconnected by the removal of cut vertices. Therefore for the normal functioning of the network even if there is parallel paths it is essential that the connectivity is greater. Now the robots navigating through the network knows its position by means of distances from all the vertices in the graph. Here we relate connectivity of graphs top fix a unique position for the robots or unique vertices in the graph that becomes basis elements. We proved general theorems regarding the choice of the basis elements whenever connectivity changes. Also mentioned special graphs and their fixation of the resolving set.

The theory of metric dimension of graphs was developed by P. J.Slater, 1975. Metric dimension of a graph is the minimum number of vertices or basis elements required for the navigation. These basis elements identify all the vertices in the graph by means of distances from it.

Molecular graph can be inferred as a mathematical graph where each atom represents nodes and the each bond between atoms represents edges. By means of the basis elements we can define another distance matrix with respect to the nodes in the graph.

PRELIMINARIES AND BASIC NOTIONS

We use all the definitions, results and notations about graphs and about metric dimension of graphs given in (Alexandrut.Balaban, 1985; Caceres *et al*, 2005; Chartrand *et al.,* 2000; Douglas B. West; Erdos, 1960; Harary *et al.,* 1976).

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Let $G = (V, E)$ be a simple connected non empty graph.

A cut vertex of a graph is a vertex $v \in V(G)$ whose removal will disconnect the graph or reduces into K_1.

The vertex connectivity of a graph is the minimum number of vertices whose removal will disconnect the graph or reduces into . It is denoted by.

A graph is said to be n-connected if and it is said to be 2-connected if and only if there are two internally disjoint paths between the distinct vertices and .

A chemical graph is a graph whose vertices are the atoms and the edges correspond to chemical bonds.

Locus of Robots with respect to different graphs

In this section we established the result that fixes the locus of the basis elements navigating in different graphs. Connectivity of graphs has great importance in order to maintain the information and basic properties in network. Here we try to connect the cut vertices and navigation of robots in the network.

RESULTS

Theorem

If $\beta(G) = 1$ and $\kappa(G) = 1$ then place the basis element on a vertex other than the cut vertex.

Proof

Since $\kappa(G) = 1$the graph Ghas a cut vertex say v. Take two arbitrary vertices u and $w$in G. Clearly every $u - w$path passes through the cut vertex v. Let there be a path say $u_1, u_2, \ldots, u, v, w, \ldots, u_n$ in G. Then obviously coordinate of the vertices u and $w$will be the same if we place the basis element on the cut vertex v. This is not possible. So place the basis element on a vertex that is not a cut vertex.

Example

In the case of P_n, $\beta(P_n) = 1$and $\kappa(P_n) = 1$. It is clear that all the vertices other than end vertices are cut vertices. So we can place the basis element on either of the two end vertices $v_1$or v_n.

Theorem

Let$|S| = \kappa(G)$, $\beta(G) = 2$and $\kappa(G) \geq 2$. Then place one of the basis element on a vertex $v \in S$and the other basis element on a vertex that is adjacent to v. Otherwise place the basis elements on the vertices that are adjacent to v.

Proof

First suppose that $S = \{u, v\}$and $e = uv$ be an edge in G. Since $\kappa(G) = 2$there is another path say $u, u_1, u_2, \dots, u_n$exists other than the edge $e = uv$. Suppose we place the basis elements on u and $v$and both u and $v$are adjacent to more than one vertices. Then those vertices have same coordinate (1,1). Now consider the case that both u and $v$are not adjacent to any of the u_i, $i = 1,2, \dots, n$then we get a cycle $C = u, u_1, u_2, \dots, u_n, v, u$$. Obviously $d(u) \geq 3$and $d(v) \geq 3$otherwise $\omega(G - (uv))$ is not greater than $\omega(G)$. The coordinate of u_1 will be (1,2) and $u_n$will be (2,1). Since $d(u) \geq 3$and $d(v) \geq 3$both u and $v$is adjacent to the vertices say v_j and v_k such that $v_j v_k \notin \{u_1, u_2, \dots, u_n\}$. Then v_j's coordinate is (1,2) and v_k's coordinate will be (2,1). That is not possible since in that case we will not obtain distinct coordinates. So place one of the basis element on one vertex in S and the other basis elements on the vertices that are adjacent to the vertices S.

Suppose$\omega(G - (uv)) > \omega(G)$.Let $S = \{v_1, v_2, \dots, v_r\}$and $W = \{u_1, u_2, \dots, u_m\}$ where $r \leq m$or $r \geq m$. If $r < m$then we can place the basis elements on the cut vertices and if $r \geq m$ and $m \geq 1$then do not place the basis elements on the cut vertices. In particular if $= 1, m \geq 1$ and we place the basis elements on the cut vertex then the adjacent vertices of that cut vertex will have same coordinates.

Example

Consider the graph below. It is clear that $\beta(G) = 2$and $\kappa(G) = 2$. So $r = m$. In the first figure it is shown that if we place the basis elements on the cut vertices namely u and v then the vertex in the graph does not have distinct coordinates. In the second figure we place the basis elements on the vertices that are adjacent to the vertices u and $v$then the all the vertices are of distinct coordinates.

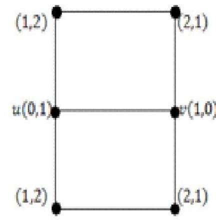


Figure 1

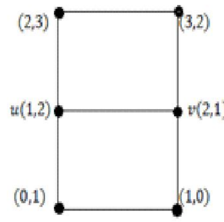


Figure 2

Theorem

If $G - S = K_1$and $|S| > 1$then place the basis element on the cut vertices.

Proof

If $G - S = K_1 = u$ (say) then u is adjacent to the vertices in S as well as the other vertices in G. Let $S = \{v_1, v_2, \dots, v_r\}$ and $W = \{u_1, u_2, \dots, u_m\}$ be the set whose removal reduces the graph to K_1 and the basis of G. The coordinate of $v_l, l = 1,2, \dots, r$ are $(a_{l1}, a_{l2}, \dots, a_{lm})$ where $a_{li} = d(v_l, v_i)$, $i = 1,2, \dots, m$. Then the coordinate of u will be $(a_{l1} + 1, a_{l2} + 1, \dots, a_{lm} + 1)$ distinct from the other vertices coordinates. Thus we can choose the vertices in S as basis elements. In particular as an optimization we consider the case of complete graph with n vertices. We know that $\beta(K_n) = n - 1 = \kappa(K_n)$ and the removal of these $n - 1$ vertices reduces $K_n$into K_1. So we can choose these $n - 1$ vertices as basis elements.

Theorem

Let the cut vertices of C_n are $u$and v. Then we can choose the two basis elements as the cut vertices in the following cases:

(i) If u is adjacent to w (not a cut vertex) then choose u and w as basis elements.

(ii) Clearly u and v are not adjacent in C_n then we have two possibilities
Case 1: If u and v are at an even or odd distance then choose u and v as basis elements if the number of vertices on the two internally disjoint $u - v$ paths are not equal.
Case 2: If $d(u, v) = 2n + 1$ or $d(u, v) = 2n$ then choose one of the basis element as u and the other as a vertex adjacent to v.

Proof

(i) In this case the coordinate of u and w are (0,1) and (1,0) respectively. Let the cycle be $u, w, v_1, v_2, \dots, v_n, u$, $n \geq 1$. If $n = 1$ we get a triangle then the coordinate of v_1 will be (1,1). If $n = 2$ then we get the coordinates of v_1 and v_2 are $r(W/v_1) =$ (2,1)and $r(W/v_2) =$ (1,2). If $n = 3$ the coordinates are $r(W/v_1) =$ (2,1), $r(W/v_2) =$ (2,2)and $r(W/v_3) =$ (1,2).

If there is a vertex v_j such that $d(u, v_j) = d(w, v_j) = m$ then the coordinate of v_j is (m, m) and the coordinates of the vertices $v_{j-1}, v_{j-2}, \dots, w$ are $(m, m-1), (m-1, m-2), \dots, (m-(m-1), m-m)$ and the coordinates of $v_{j+1}, v_{j+2}, \dots, u$ are $(m-1, m), (m-2, m-1), \dots, (m-m, m-(m-1))$ respectively.

If there is no such v_j so that $d(u, v_j) = d(w, v_j)$ then we can find two adjacent vertices v_j and v_{j+1} for which v_j is the vertex at distance m from u and $m-1$ from w and v_{j+1} is the vertex at a distance $m-1$ from u and m from w. Then the coordinate of $v_{j-1}, v_{j-2}, \dots, w$ are $(m-1, m-2), (m-2, m-3), \dots, (m-(m-1), 0)$ and the coordinates of $v_{j+1}, v_{j+2}, \dots, u$are $(m-2, m-1), (m-3, m-2), \dots, (0, m-(m-1))$. Thus in any case we have distinct coordinate for all the vertices if we choose u and w as basis elements.

Proof(ii)

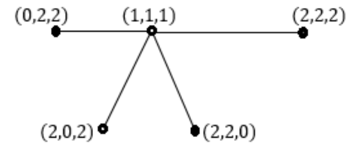
Suppose u and v are at an even distance and the number vertices on two internally disjoint paths are not same. Let $d(u, v) = 2k$ then the coordinates of u and v are (0,$2k$)and ($2k$,0). Let the internally disjoint $u-v$ paths from u to v are $P = u, u_1, u_2, \dots, u_p, v$ and $Q = u, v_1, v_2, \dots, v_q, v$ where $p + q = n - 2$and $p \neq q$.

Suppose p is even. Let $p = 2n$. In this case between u and v there are two middle vertices u_j and u_{j+1} with coordinates $(n, n+1)$ and $(n+1, n)$ respectively. Then the coordinates of $u_{j-1}, u_{j-2}, \dots, u_1$ are$(n-1, n), (n-2, n-1), \dots, (n-1, n+1), \dots, (1, \min(n, d(u, u_1))$ and the coordinates of $u_{j+2}, u_{j+3}, \dots, u_n$are $(n+1, n-1), \dots, (\min(n, d(u, u_n), 1)$. Thereby all coordinates are distinct. Similarly if q is even we get the coordinates of the vertices on Q are distinct.

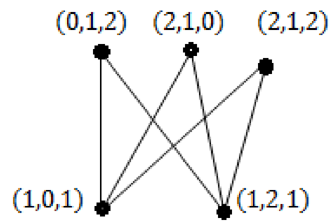
Suppose p is odd. Let $p = 2n + 1$. In this case between u and v there is middle vertex say u_j with coordinate $(n+1, n+1)$. Then the coordinate of $u_{j-1}, u_{j-2}, \dots, u_1$ are $(n, 2n-1), (n-1, 2n-2), \dots, (1, n)$ and the coordinate of $u_{j+1}, u_{j+2}, u_{j+3}, \dots, u_n$ are $(2n-1, n), (2n-2, n-1), \dots, (n, 1)$. Thus all the vertex coordinates are distinct. Similarly if q is odd then we get the coordinates of the vertices in Q are distinct.

Observations

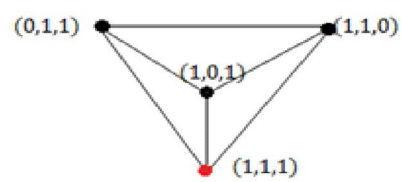
1. In a tree Tif $K_{1,m}$, $m > 1$ contains where m is the least positive integer then $\beta(T) = m - 1$. We know that $\kappa(T) = 1$. So by theorem choose the basis elements as vertices adjacent to the cut vertex.



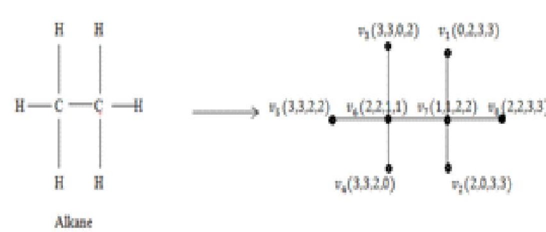
2. We know that $\beta(K_{m,n}) = \max\{m, n\}$and $\kappa(K_{m,n}) = \min\{m, n\}$. If $m, n \geq 1$ then place one of the basiselements on the cut vertex and the other basis elements are the vertices that are adjacent to the cut vertices.

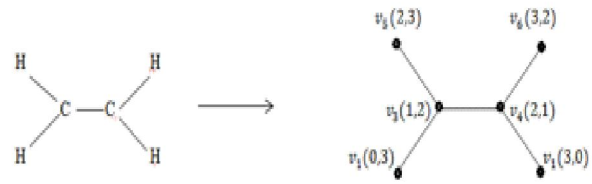


3. For the wheel graph W_n for $n > 3$ the cut set contains S contains three vertices and the removal of S reduces $W_n$toK_1. So the basis elements can be taken as the vertices in S.



4. The following are types of hydrocarbons and its molecular graph. The figure gives the coordinates of the vertices in the corresponding molecular graphs with respect to the basis elements.





Alkene

Alkyne

CONCLUSION

Fixing a unique place or choosing a unique vertex as the basis element is significant for solving a problem in navigation. In this paper we proved some results on the choice of basis elements and some applications. Further study is required to investigate the properties of hydrocarbons by means of metric dimension of graphs.

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